Can nonexistent mathematical objects make a difference? Meinongianism, indispensability argument and mathematical entanglement

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Abstract: One of the arguments that most influence the debate on the existence of mathematical objects is undoubtedly the indispensability argument. Central to this argument is the Quinean ontic thesis that we are committed to the existence of all the entities we (indispensably) quantify over in our best scientific theories. But what if a different meta-ontological paradigm is adopted? In this paper, I propose a Meinongian interpretation of the indispensability argument. The new reading of the indispensability argument, in accordance with heavy duty platonism, allows me to introduce a new notion of metaphysical dependence that goes by the name of *mathematical entanglement*, and to conclude that nonexistent mathematical objects make a difference to the concrete, physical world.

Keywords: Nonexistent Objects, Mathematical Objects, Meinongianism, Indispensability Argument, Mathematical Entanglement

Introduction

Are there any mathematical objects, such as numbers, sets, or functions? Platonists say "yes", nominalists say "no". According to Alan Baker (2003: 263), the dispute over the existence or nonexistence of mathematical objects does not stem "from any explicit thesis of platonism but from a certain background picture" inherited from traditional platonism. This picture portrays two separate worlds – the concrete world of physical objects and the abstract world of mathematical objects. Again according to Baker, this picture has directed philosophical literature to the claim that the existence of mathematical objects *makes no difference* to the concrete world:

> Since mathematical objects are acausal, the existence or non-existence of mathematical objects makes no difference to the actual arrangement of concrete objects (Cornwell 1992: 80).

That conception, as Bernard Linsky and Edward Zalta (1995) have pointed out, has led many philosophers to conceive of abstract objects *on the model* of concrete objects. Alongside this error, an arguably even more serious one has been made: the indispensability argument (IA) was found to be in harmony

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with one meta-ontological picture, i.e. Quinean meta-ontology. This error has steered IA towards a "flat"[1] image of ontology and, even more seriously, has made it so difficult to make progress in the debate over IA. In this paper, I will show how it is possible to apply a meta-ontological picture other than the Quinean one to the indispensability argument, i.e. Meinongian meta-ontology. What will result is that: *i*) the real merit of IA is not to establish whether mathematical objects exist, but to point out that mathematical objects make a metaphysical contribution to our best scientific theories; *ii*) our best scientific theories cannot be *mathematical-objects-free; iii*) there is a *mathematical entanglement* within our best scientific theories; *iv*) nonexistent mathematical objects *make a difference* to the concrete world.

Section 1. introduces and discusses Meinongian meta-ontology and, in particular, *strong Meinongianism*. Section 2. proposes a Meinongian reading of IA. Section 3. introduces the notion of *mathematical entanglement* to oppose the idea that the existence of mathematical objects "makes no difference" to the concrete world. Section 4. stresses that the notion of mathematical entanglement is in accordance with a recent form of Platonism that goes by the name of heavy duty platonism (HDP), and also, HDP is in line with Priest's Meinongianism vision of the applicability of mathematics. Section 5. concludes by focusing on the fact that mathematical objects *make a difference* to the concrete world even if they are nonexistent objects.

1. *Meinongian meta-ontology and strong Meinongianism*

The debate between Quineans and Meinongians has a long history, and most of its key episodes antedate the recent, burgeoning development of *meta-ontology*.[2] The mainstream Quinean meta-ontology claim to what has been called "the question of ontology" is:

(Q) Everything exists, because it makes no sense to speak of "nonexistent objects".

That is the apparently *easy* and *tautological* reply to that question: "What exists?". In his introduction to ontology Achille Varzi writes:

[1] The attribute "flat" is used by Jonathan Schaffer to indicate the structure of Quinean metaphysics: "For the Quinean, the target is flat. The task is to solve for E = the set (or class, or plurality) of entities. There is no structure to E. For any alleged entity, the flat conception offers two classificatory options: either the entity is in E, or not" (Schaffer 2009: 354).

[2] The term "meta-ontology" was introduced by Peter van Inwagen (1998). For van Inwagen the key question for meta-ontology is "What do we mean when we ask "What is there", and "What is the correct methodology of ontology?". A comprehensive introduction to the topic is (Berto, Plebani 2015).

As Quine has written, everything exists because it makes no sense to speak of "nonexistent entities", and those who think otherwise would manifest, not an ontological disagreement, but a misunderstanding of the very concept of existence. […] Precisely because it would be inconsistent to claim that something does not exist, however, to claim that everything exists is tautological, that is, devoid of content, therefore of interest (Varzi 2005: 3, my translation).

On the contrary for a Meinongian, pace Quine, there are objects that *do not exist*:

(M) There are things that do not exist.

Meinongianism was originally defended by Alexius Meinong (1904), and the view has recently been developed and defended by Terence Parsons (1980), Richard Routley (1980), Edward Zalta (1988), Graham Priest (2005) and Francesco Berto (2013; 2015). The order of the names above is not casual but corresponds to a precise way of understanding the Meinongian *Principle of Comprehension*. Indeed, any Meinongian theory needs some principle stating which objects are admitted by the theory, and which properties they can have. In its naïve version Meinongianism defends to what Parsons (1980) called an "Unrestricted Comprehension Principle" (UCP) for objects:

(UCP) For any condition $A[x]$, with free x, some object satisfies $A[x]$.

UCP is the foundation of the naïve version of Meinongianism, which holds that for every given set of properties there is some object that exactly bears those properties. Last century, this view was widely criticised by Bertrand Russell (1903; 1905a; 1905b) and by Willard Van Orman Quine (1948) on three several counts:[3]

1) the property of existence[4] can be added to any combination of properties to yield existent objects that are incomplete (e.g. an existent object that is green and nothing else);
2) the existence of inconsistent object (e.g. round square), or that straightforwardly disagrees with empirical observation (e.g. golden mountain on Earth);
3) it is not clear how an adequate criterion of identity can be given for the domain of nonexistent objects.

[3] On why there is no contradiction in the Meinongian thesis and why the Quinean thesis is not clear see (Sendłak 2023).

[4] Of course, it only makes sense to talk of "properties of existence" from a Meinongian point of view or for those who defend a *classical* position of existence and essence. From Frege to Quine and for many philosophers today, existence is not a property. For an analysis of the limitations of Frege-Quine's quantificational theory of existence see (Galvan 2015), while for a new system of logic for existence and essence see (Galvan, Giordani 2020).

Since many of the objections to Meinongianism have focused on the Unrestricted Comprehension Principle, in the last last few decades different approaches to the problem have been developed. One approach, Nuclear Meinongians, limited UCP to a restricted vocabulary and distinguished between two kinds of predicates, called *nuclear* and *extranuclear* (Parsons 1980; Routley 1980). The second approach, by dual copula Meinongians, distinguishes between two ways in which things can be ascribed properties: ordinary predication expressing property-instantiation and *encoding* (Zalta 1983; 1988). Finally, the third approach is far more recent, to be called Modal Meinongianism, and claims that an object has all the properties it is characterized as having not necessarily in the actual world but in some possible or impossible worlds (Priest 2003; 2005; Berto 2013; 2015; Berto, Priest 2014). Regardless of which approach of neo-Meinongianism one wishes to defend, one can correctly say that Meinongianism is understood as "the view that some objects do not exist, but we can generally talk about them, quantify on them, and state true things about them" (Berto 2013: 115). I agree with Filippo Casati and Naoya Fujikawa that, given this definition, contemporary Meinongians support the following three ideas:

[1] Some objects do not exist.

[2] We can refer to and quantify over nonexistent objects to make true statements about them.

[3] Nonexistent objects are (or, at least, are parts of) the truth-makers of sentences which contain reference to or/and quantification over nonexistent objects.

Casati and Fujikawa call a position which endorses [1], [2] and [3] *strong Meinongianism*:

> And contemporary Meinongians like Parsons, Routley, Zalta and Priest seem to take the property-possession of nonexistent objects as contributing to truth-making of sentences about nonexistent objects. For example, even if a is a nonexistent object, if a has the property P, then, the sentence 'a is P' is true simply because of a's possession of P. In this way, nonexistent objects can be involved in truth-making of sentences about them. Moreover, Meinongianism accepts that a nonexistent object can have properties which are not parts of its characterization. […] Because of this, according to these contemporary Meinongians, nonexistent objects can be involved in the truth-makers of sentences such as 'Sherlock Holmes is possible' and so on. Let us call a position which endorses [1], [2] and [3] strong Meinongianism. Many contemporary Meinongians seem to endorse strong Meinongianism (Casati, Fujikawa: 424-425).

The strength of this position, which will play a key role in this paper, is that it overcomes some of the problems of *reductionist approach* regarding at the instantiation of the property of being possible by nonexistent objects, and "the tension between admitting quantification over nonexistent objects, on the one hand, and the reductionist approach which tries to eliminate nonexistent objects from truth-making, on the other hand" (*ivi*: 433). The solution, according to Casati and Fujikawa, is to adopt strong Meinongianism by admitting truth-making by nonexistent objects.

2. *The indispensability argument*

Generally, the indispensability argument was understood as an argument for the existence of abstract mathematical objects (Colyvan 2001; Panza, Sereni 2013; Cuconato 2022). For those who support IA we should believe in mathematical objects for the same reason as we believe in microphysical particles: because our best scientific explanations imply their existence. In the words of Hartry Field:

> [I]f our belief in electrons and neutrinos is justified by something like inference to the best explanation, isn't our belief in numbers and functions and other mathematical entities equally justified by the same methodology? After all, the theories that we use in explaining various facts about the physical world not only involve a commitment to electrons and neutrinos, they involve a commitment to numbers and functions and the like (Field 1989: 16).

This suggests that the most direct formulation of the argument should be something like this (Liggins 2016: 532):

i) Mathematics is indispensable to science: that is, our best explanations imply the existence of numbers and other mathematical objects;

ii) If mathematics is indispensable to science, then there are mathematical objects;

[I] ————————————

iii) There are mathematical objects.

Another of the most debated formulations of IA has been advanced by Mark Colyvan (2001: 11):

i) We ought to have ontological commitment to all and only those entities that are indispensable to our best scientific theories;

ii) Mathematical entities are indispensable to our best scientific theories;

[II] ———————————

iii) We ought to have ontological commitment to mathematical entities.

According to Otávio Bueno "the indispensability argument crucially relies on the conditions under which quantification over certain objects is indispensable" (Bueno 2018: 203). There are three conditions identified by Bueno (*ivi*: 203-204) and they correspond to three distinct roles played by mathematical objects in scientific theories:

a) Expressive role: Quantification over certain objects is, in many cases, made in order to express certain facts (situations, possibilities, etc.). If such expressions cannot be formulated without reference to the objects in question, the quantification is expressively indispensable.
b) Predictive role: Objects are often quantified over in predictive contexts in which they are invoked to make predictions about certain phenomena. If such predictions cannot be obtained without reference to the relevant objects, the quantification is predictively indispensable.
c) Explanatory role: Objects are frequently quantified over to explain certain phenomena. If the explanations in question cannot be implemented without reference to such objects, the quantification is explanatorily indispensable.

In turn, the conditions under which quantification over certain objects is indispensable are based on the standard view of existence, i.e. Quine's criterion for ontological commitment. I will call these forms of IA *standard indispensability argument* (S-IA). Standard because the standard view of existence is applied, i.e. Quine's criterion for ontological commitment. Generalizing, Quine's criterion of ontological commitment can be formulated as follows:

(QC) An object *o* is an ontological commitment of a regimented (set of/) statement(s) iff *o* is required to make the statement(s) true (where *o* is assumed to be in the range of the bound variable(s)).

Quine puts it this way: "To be assumed as an entity is…to be reckoned as the value of a variable", and "we look to bound variables in connection with ontology not in order to know what there is, but in order to know what a given remark or doctrine…says there is" (Quine 1948: 13). A contemporary Quinean is Peter Van Inwagen (1998). Van Inwagen sums up Quine's position in a series of five theses:

1. Being is not an activity.
2. Being is the same as existence.
3. Existence is univocal.
4. Existence is expressed by the existential quantifier.

5. (QC) is a procedural norm for ontological disputes.

Let's focus on thesis 5.[5] According to Ted Parent (2019) "(QC) lays down one of the 'rules for engagement' for ontological disputes".[6] It is precisely this rule for engagement that has tied the debate on the subject of indispensability almost exclusively to Quinean meta-ontology. In this sense, it's fair to say that QC is the *indispensable meta-ontological premise* to the standard version of IA. But what if we read the indispensability argument through the glasses of a Strong Meinongian? The structure of IA would not change. What would totally change is the way we understand that *there are mathematical objects*. For a Meinongian to say that there are objects of type does not mean that objects of type exist. Not only that, if we accept the position of the majority of neo-Meinongians as Parsons, Routley, Zalta, Priest or Berto, mathematical objects are abstract objects and "abstracts objects are just kind of non-existent object" (Priest 2005: 135):[7]

[4] Mathematical objects are nonexistent objects.

If we replace QC with with [1], [2], [3], [4] the most direct formulation of IA should be something like this:

i) Mathematics is indispensable to science: that is, our best explanations imply that *there are* numbers and other mathematical objects;
ii) If mathematics is indispensable to science, then *there are* mathematical objects;

[III] ———————————

iii) *There are* nonexistent mathematical objects.

iv) Nonexistent mathematical objects are the truth-makers of sentences which contain reference to or/and quantification over mathematical objects.

I will call this form of IA *non-standard indispensability argument* (NS-IA). Non-standard because expressions such as "there are" must be understood in the sense of "Meinongian quantification". To say that "there are nonexistent mathematical objects" is not to say that such objects exist. In this way, the focus is shifted from "rules for engagement-existence" to "nonexistent objects-truthmaker". The argument of indispensability thus developed without

[5] More precisely thesis 5 is not given a one-sentence formulation in van Inwagen; but he says that it is really a "family of theses" (2014: 85).

[6] Not by chance, van Inwagen illustrates this using the Platonism vs. nominalism debate about numbers.

[7] It is interesting to note that for Meinong (1904) abstract objects do not exist, but they do *subsist*.

denying the conditions under which quantification over certain objects is indispensable, allows us to conclude that "there are" mathematical objects, but we are not ontologically committed to them – at least in the Quinean sense of ontological commitment. What I want to argue is that the cause of the stalemate in the IA debate was precisely Quine's quantificational theory of existence. In this way, the debate was regimented on the topic "*rules for engagement-existence-indispensability*".

Also [III], like [I] and [II], is based on the three conditions under which quantification over certain objects is indispensable, but in NS-IA changes the way quantification over mathematical objects is understood to express, predict, and explain a variety of features of the world. If Quine "used the indispensability argument as a reason to support ontological commitment to classes" (Bueno 2018: 204), in contrast, a meta-ontological perspective such as Meinongianism, allows us to consider IA valid even if mathematical objects do not exist and to use IA as a reason to support the metaphysical role that mathematical objects play in our best scientific theories. NS-IA admits truth-making by nonexistent objects, and therefore, the strength of analysis of NS-IA, is to shift the focus to the fact that mathematical objects *make a difference* to the concrete world even if they are nonexistent objects.

3. *Orthogonality and mathematical entanglement*

In recent years, the scientific literature on IA has developed a critique inspired by the idea that the existence of mathematical objects "makes no difference" (MND) to the concrete, physical world. Baker (2003) provides effective critiques of some ways in which "makes no difference" claims have been defended. I totally agree with Baker (2003) that there is a tight connection between IA and "makes no difference" claims, however, I prefer Matteo Plebani's formulation of MND. Here is how Plebani (2018: 256) proposes to understand MND claims:

– (Orthogonality) The two subject matters how the concrete world is and whether there are abstract objects are orthogonal.

In detail, Plebani referring to Lewis' original definition of subject matter, and the works of Stephen Yablo (2012; 2014), understands orthogonality "as the view that the way the concrete world is does not demand nor preclude the existence of abstract mathematical objects (Yablo 2012): given a world in which mathematical objects are present (absent) there is a world *w'* in which mathematical objects are absent (present) and that is concretely indiscernible from *w"* (Plebani 2018: 257).

Nominalists, relying on the notion of causal dependence, have been attracted by orthogonality:

– (BAD-ProO) Mathematical entities have no causal powers; therefore the existence of mathematical entities makes no difference for how the concrete world is.

Nevertheless, as Baker (2003) points out, this argument for orthogonality is unsatisfactory because (BAD-ProO) is fundamentally based on a certain background picture inherited from traditional platonism: mathematical objects have no causal powers and therefore "mathematical objects make no difference for the concrete world from the fact that mathematical objects make no difference for the concrete world at the causal level" (*ivi*: 257). Those who defend (BAD-ProO) do not take into account that there are other forms of dependence besides causal dependency, e.g. metaphysical dependence.

Alongside (BAD-ProO) Plebani also presents a bad argument developed by Yablo:

– (BAD-AntiO) "we cannot imagine-without-numbers a complex world" therefore "we cannot imagine a complex world lacking in numbers" (Yablo 2012: 1014).

Those who defend (BAD-AntiO) write Plebani "confuses the issue of what it takes to describe certain circumstances with the issue of what it takes for those circumstances to obtain" (Plebani 2018: 257). In general, I agree that (BAD-AntiO) is not such a strong argument (although I find the issue of "what it takes" quite obscure). However, what I am interested in proposing now is not a criticism or a defence of the two positions mentioned above, but rather to present a new argument against orthogonality.

One of the key points of my paper is the distinction between an MND developed from an *ontological* point of view and an MND developed from a *metaphysical* point of view. The distinction between the ontological and metaphysical planes is the result of the Meinongian thesis that some things, that is, some property-bearers or *Sosein*-bearers, are nonexistent or lacking *Sein*. This thesis is the famous Meinongian *Principle of Independence* states that an object's *Sein*, its existential status, is independent from its *Sosein*, its having properties.[8] Given

[8] It is important to specify that the Principle of Independence: *i*) is connected by Meinong to the Principle of the *Aussersein*, or "principle of the indifference of pure Objects to being" (Meinong 1904: 86): being and non-being are not part of the Sosein of the thing; and *ii*) is set against a very strong metaphysical thesis: called *serious actualism*, consisting in the thesis that the having of whatever property entails existence. Serious actualism has deep roots in the history of philosophy (Berto 2013: 86) but it is especially in the actualism of Russell the idea that the having of any property entails existence. Quinean meta-ontology was influenced by Russellian actualism and, consequently, serious actualism directed IA and its subsequent debate. Finally, it is historically and theoretically important

the Principle of Independence, to say that an object *X* has the property of makes a difference to the concrete world, does not also mean that objects *X* exist.

In the previous section, I highlighted the weight that the choice of a particular meta-ontological picture occupies in IA. In particular, I interpret Quinean "regimentation" as the cause of the stalemate in the IA debate: we translate a scientific theory into classical first-order logic, then read off its ontological commitments as per (QC), we will have a criterion for establishing what objects such a theory says exist. In a Meinongian perspective IA does not establish that mathematical objects exist, but rather that mathematical objects contribute to the truth-making of sentences concerning concrete objects. Therefore, given [III], it is possible to reformulate MND by placing the emphasis no longer on the ontological aspect (there are abstract objects) but on the metaphysical one:

– (New Orthogonality) The two subject matters how the concrete world is and whether there are abstract objects that make a metaphysical contribution to our best scientific theories are orthogonal.

The real merit of the indispensability argument and "makes no difference" claims is not to establish whether mathematical objects exist or not exist, but whether mathematical objects make a metaphysical contribution to our best scientific theories. In this sense, [III] brings out the indispensability of mathematical objects in the sense that our best scientific theories cannot be *mathematical-objects-free* because: *i*) mathematics plays a rolee expressive, inferential, and explanatory in the scientific practice and; *ii*) mathematical objects contribute to the truth-making of sentences concerning concrete objects. In other words, to say that our best scientific theories cannot be mathematical-objects-free is to say that there is an inextricable *mathematical entanglement* within our best scientific theories. The term "entanglement" is inspired by *quantum entanglement* and was coined by Erwin Schrödinger in 1935. John Clauser, Anton Zeilinger and Alain Aspect have won the 2022 Nobel Prize in Physics for their experiments with entangled photons, in which particles

to specify that the philosophical roots of the principle of independence are well established in Scholastic metaphysics in the doctrine of the *real distinction* between the *essence* of the thing (that is to say, what the thing is) and the *existence* of the thing (that it is) in authors such as Avicenna, Duns Scotus and especially Thomas Aquinas. As Edward Feser points out: "A third argument (with Aquinas presents in Chapter IV of *On Being and Essence*) holds that we can know the essence of a thing without knowing one way or the other whether it exists. Suppose a person had never before heard of lions, velociraptors, or unicorns, and you give him a thorough description of the nature of each. You then tell him that of these three creatures, one exists, one used to exist but is now extinct, and the third never existed; and you ask him to tell you which is which given what he now knows about their essences. He would, of course, be unable to do so. But then existence of the creatures that exist must be really distinct from their essences, otherwise one could know of their existence merely from knowing their essences" (Feser 2014: 243).

of light become *inextricably linked.*[9] In the same way as quantum particles, physical objects are inextricably linked to mathematical objects. Because of the effects of mathematical entanglement, the properties of an abstract mathematical object influence the properties of a concrete physical objects. This mathematical entanglement makes it clear that the relations between abstracta and concrete are metaphysically fundamental and they do not hold *in virtue of* any nominalistic properties possessed by the concrete objects alone.[10] Based on this, it is possible to formulate a good argument against (new) orthogonality:

– (GOOD-AntiO): since mathematics plays a rolee expressive, inferential, and explanatory in scientific theorizing and mathematical objects contribute to the truth-making of sentences concerning concrete objects, our best scientific theories cannot be mathematical-objects-free; therefore, our best scientific theories are committed to *mathematical entanglement*; the relation between abstracta and concreta are metaphysically fundamental; and mathematical objects "make a difference" to the concrete, physical world.

In this way, on the one hand, Meinongian meta-ontology makes it possible to avoid commitment to mathematical objects despite granting that quantification over them is indeed indispensable to our best theories of the world, on the other hand, (GOOD-AntiO) shows that the concrete world demands abstract mathematical objects.[11] This seemingly contradictory *phenomenon* is possible because mathematical objects should not be classified as pure *fictional objects*:[12] "things mentioned and described in tales, novels, fantasy stories, operas, and so on" (Berto, Plebani 2015: 104). Indeed, while it is true that existence should not be required for successful reference or quantification to objects that do not exist,[13] such as Sherlock Holmes or the number 2, it is also true that the relationship that our scientific theories have with nonexistent objects is profoundly

[9] This phenomenon leads to paradox and has puzzled physicists for a long time because the behavior of entangled particles is apparently inexplicable and incomprehensible. The trio's experiments proved that connections between quantum particle comes from a genuine association in which manipulating one quantum object affects another far away. Albert Einstein famously called the phenomenon "spooky action at a distance" – it is now known as quantum entanglement.

[10] In this sense, the notion of "mathematical entanglement" is related to the notion of "grounding". The *grounding theory* refers to a relation of non-causal dependence, or metaphysical dependence, generally introduced by the use of terms like "in virtue of" or "because". See Liggins (2016) for a nominalist strategy developed through the connection between grounding and IA.

[11] An argument from the presumption that mathematics is indispensable for science to the claim that mathematical objects make a difference to the concrete world is developed by Colin Cheyne and Charles Pigden (1996).

[12] In general, for a study of the advantages of Meionongianism over fictionalism see (Gan 2021).

[13] Bueno (2018: 215) argues in detail how it is possible to successfully refer to objects that do not exist or successfully quantifies over objects that do not exist.

different depending on the *type* of nonexistent object. Consider, for instance, complex numbers. Generally, the number system we all know, like 2, -5, 0.8, √2, 16/4, and π, are known as the real numbers ℝ, while, a complex number is a number written in the form:

$$z = a + bi$$

where *a* and *b* are real numbers.[14]

Complex numbers have many applications in physics, chemistry, biology, electrical engineering, statistics, and economics. In particular, they have become fundamental in quantum mechanics:

> Quantum mechanics seems to use complex numbers in a more fundamental way. It suffices to look at some of the most basic equations, both in the matrix ($[\hat{p}, \hat{x}] = ih$) and wave ($(ih(\partial\psi/\partial t = \hat{H}\psi)$) formulations, to wonder about the presence of the imaginary unit. This complex nature of quantum mechanical quantities puzzled some of the very founders of the theory (Karam 2020: 39).

It becomes clear that complex numbers play a fundamental role in scientific practice and contribute to the truth-making of sentences concerning concrete objects. In contrast, for other nonexistent objects, such as Sherlock Holmes, Harry Potter or Alice in Wonderland, their metaphysical role and contribution in scientific theorizing cannot be traced.[15] Therefore, complex numbers are inextricably linked to physical objects and make a metaphysical difference to the way the concrete world is. One moral to be drawn from (GOOD-AntiO) is that the concrete world demands abstract mathematical objects, and the relations between abstracta and concrete are not *grounded* in the nominalistic properties of concrete objects.[16]

4. *Heavy duty platonism*

The last aspect I want to address is how the concept of mathematical entanglement is inherent in a recent form of Platonism that goes by the name of *heavy duty platonism* (HDP). Also, HDP is in line with Priest's Meinongianism vision of the *applicability of mathematics*. Let's proceed in order. HDP is a view about the metaphysics of physical magnitudes, like temperature and mass. Robert Knowles characterizes HDP as "the view that physical magnitudes, such as mass and tem-

[14] *a* is known as the "real part" of *z*, and *b* as the "imaginary part". Furthermore, we define the imaginary unit number *i* as the square root of -1, that is: $i = \sqrt{(-1)}$.

[15] In this way, the metaphysical structure of nonexistent objects is not *flat*.

[16] This aspect differentiates my position from mathematical fictionalism.

perature, are cases of physical objects being related to numbers" (Knowles 2015: 1255). The key point is how HDP understands this "being related":

> what they tell us about these relations: weaker forms tell us they are derivative of more fundamental properties or relations that hold of physical objects alone, while HDP says these relations are fundamental (*ivi*: 1256).

One of the most interesting arguments formulated by Knowles is based on the Lewisian distinction between intrinsic and extrinsic properties:

> A thing has its intrinsic properties in virtue of the way that thing itself, and nothing else, is. Not so for extrinsic properties, though a thing may well have these in virtue of the way some larger whole is. The intrinsic properties of something depend only on that thing; whereas the extrinsic properties of something may depend, wholly or partly, on something else (Lewis 1983: 111-112).

Knowles attacks the intuition that some physical magnitude properties (such as mass) are clear-cut cases of intrinsic properties. Those who defend this insight believe that an object's mass involves only that object; and since HDP implies that an object has its mass by being related to something external such as a number, HDP implies mass is extrinsic and, therefore, HDP is false. One of the arguments in defence of HDP is to show how the intuitions we have about the properties of physical phenomena are unreliable. In the Standard Model – our best physical theory of fundamental particles – the mechanism by which a particle gains its mass is based on the assumption of the existence of a scalar field entirely distinct from any particle, the Higgs Field:

> The mass of particles depends not only on the properties of the particles, but also on the properties of the Higgs Field. The Higgs Field is entirely distinct from any particle interacting with it. For any particle *p*, there is a world in which *p* does not exist while the Higgs Field does. Science tells us that the mass of a particle depends on the properties of something distinct from the particle, so, on the present view, mass is extrinsic. According to HDP, then, mass is a three-place relation holding between an object, a number, and the Higgs Field (Knowles 2015: 1260).

This fundamental form of the relationship between abstracta and concrete is precisely expressed by *mathematical entanglement*. To say that there is a mathematical entanglement within our best scientific theories, means to say from the point of view of HDP "that there is a robust metaphysical connection between physical objects and numbers that renders the latter explanatorily relevant to physical phenomena" (*ibidem*). It is the mathematical entanglement that offers a possible explanation for why such abstract objects as complex numbers make a difference to the concrete world:

> Complex numbers seem to be fundamental for the description of the world proposed by quantum mechanics. In principle, this can be a source of puzzlement: Why do we need such abstract entities to describe real things? (Karam 2020: 43).

We need the complex numbers to describe "real things" because there is a solid metaphysical connection between physical objects and mathematical objects, that is, between existent objects and nonexistent objects, that is, between causal entities and non-causal entities.

But how can mathematical entanglement and HDP help in the question of the applicability of mathematics? Priest (2003) addresses the problem by asking the question from a Meinongian point of view: "How can non-existent objects tell us anything about existent ones?" (*ivi*: 11). Priest after referring to Routley's proposed solution goes straight to the heart of the problem:

> How, then, is one to explain the fact that properties of non-existent objects can tell us something about existent objects? Actually, exactly the same question can be posed for platonism, and the answer in both cases is the same. The physical quantities in question have certain properties, and the mathematical quantities have other properties. But we can move between the one and the other because these properties have the same structure, and, specifically, because the correlation established by the bridge laws is an isomorphism (*ivi*: 12).

According to Priest we can move between physical and mathematical properties because these properties have the same *structure.*[17] And this structure is exactly the robust metaphysical connection between concrete physical objects and mathematical abstract objects that HDP subscribes to. In this way, the fact that the properties of nonexistent objects can tell us something about existent objects is explained by the fact that existent objects being metaphysically related to nonexistent objects. And that is why our best scientific theories explaining how the concrete world is made are instances of mathematical entanglement. For this, Priest's meinongianism differs from weak versions of platonism because according to HDP platonic relations between nonexistent objects and existent objects are metaphysically fundamental. In the words of Knowles:

> It is quite plausible to hold that some physical objects have some of their causal powers by instantiating relations to non-causal entities (Knowles 2015: 1267).

Within this framework, at least two fundamental questions remain unanswered and will have to be investigated in the future. The first, concerns the form of mathematical structuralism that might be in accordance with NS-IA

[17] Mathematical structuralism is the view that pure mathematics is about abstract structure or structures.

and HDP. In general, I think one must investigate those forms of structuralism that "acknowledges that abstract structures exist, that the pure objects of mathematics are in some sense elements of, or places in, those structures, and that there is nothing more to the pure objects of mathematics than can be described by the basic relations of their corresponding structure" (Nodelman, Zalta: 39).[18] The second, concerns the possibility of defining a rigorous metaphysical justification for mathematical entanglement within the model of meta-metaphysics developed by Alessandro Giordani and Ciro De Florio (2020).[19]

5. *Conclusion*

The present paper has shown the following: first, the real merit of IA is not to establish whether mathematical objects exist, but to point out that mathematical objects make a difference to the concrete world. Secondly, our best scientific theories cannot be mathematical-objects-free is because there is a mathematical entanglement within our best scientific theories. Thirdly, the fact that the properties of nonexistent objects can tell us something about existent objects is explained by the fact that existent objects being metaphysically related to nonexistent objects. Therefore, the strength of NS-IA lies in shifting the focus to the statement that mathematical objects make a difference to the concrete world even if they are nonexistent objects.

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[18] On this form of structuralism see (Dedekind 1888; Parsons 1990; Shapiro 1997) and in particular mathematical structuralism of Uri Nodelman and Zalta. Their structuralism is based on the following insights: "First, […], abstract objects are connected to the properties that define them in a different way than ordinary objects are connected to the properties they bear. Second, […], theorems and truths about abstract relations are more important in defining mathematical structures than mathematical entities" (Nodelman, Zalta 2014: 40).

[19] Giordani and De Florio proposed a new meta-metaphysics model (*Metaphysics Constrained by Physics or Logic*) within which they acknowledge: *i*) an autonomous field of enquiry for metaphysics; and *ii*) the possibility for metaphysics to be based on justification procedures that are "in part different from the empirical procedures of science or the a priori procedures of logic" (De Florio, Giordani 2020: 948, my translation).

References

Baker, Alan, 2003, "Does the existence of mathematical objects make a difference?", in *Australasian Journal of Philosophy*, 81, 2: 246-264.

Berto, Francesco, 2013, *Existence as A Real Property: The Ontology of Meinongianism*, Springer, London.

—, 2015, "There Is an 'Is' in 'There Is': Meinongian Quantification and Existence", in Torza, Alessandro (ed). *Quantifiers, Quantifiers, and Quantifiers: Themes in Logic, Metaphysics, and Language*, Synthèse Library, vol 373, Springer, Cham: 221-240.

—, Matteo Plebani, 2015, *Ontology and Metaontology. A Contemporary Guide*, Bloomsbury.

—, Graham Priest, 2014, "Modal Meinongianism and Characterisation: Reply to Kroon", in *Grazer Philosophische Studien*, 90, 1: 183-200.

Bueno, Otavio, 2018, "Putnam's indispensability argument revisited, reassessed, revived", in *Theoria. An International Journal for Theory, History and Foundations of Science*, 33, 2: 201-218.

Casati, Filippo - Naoya Fujikawa, 2016, "Nonexistent Objects as Truth-Makers: Against Crane's Reductionism", in *Philosophia*, 44: 423-434.

Cheyne, Colin, Charles R. Pigden, 1996 "Pythagorean Powers, or A Challenge to Platonism", in *Australasian Journal of Philosophy*, 74, 4: 639-45.

Colyvan, Mark, 2001, *The Indispensability of Mathematics*, Oxford University Press, Oxford-New York.

Cornwell, Stuart, 1992, "Counterfactuals and the Applications of Mathematics", in *Philosophical Studies*, 66: 73-87.

Cuconato, Simone, 2022, *Impegno ontologico e l'argomento di indispensabilità in filosofia della matematica*, Il Sileno Edizioni, Lago (CS).

—, 2024, "Fol-Based Applied Ontology for Metadata Extraction in Mathematical Knowledge Management", in *Romanian Journal of Mathematics and Computer Science*, 14, 1: 1-11.

Dedekind, Richard, 1888, *Was sind und was sollen die Zahlen*, Vieweg und Sohn, Braunschweig.

De Florio, Ciro, Alessandro Giordani, 2020, "Alcune ipotesi sulla possibilità della metafisica", *Rivista di Filosofia Neo-Scolastica*, 4: 943-967.

Feser, Edward, 2014, *Scholastic Metaphysics. A Contemporary Introduction*, editiones scholasticae, Heusenstamm.

Field, Hartry, 1989, *Realism, mathematics and modality*, Blackwell, Oxford.

Galvan, Sergio, 2015, "Quantificatori ed esistenza", in *Rivisita di Filosofia Neo-Scolastica*, 1-2: 143-153.

—, Alessandro Giordani, 2020, "A Classical Logic of Existence and Essence", in *Logic and Logical Philosophy*, 29, 4: 541-570.

Gan, Nathaniel, 2021, "Fictionalism and Meinongianism", in *Theoria. An International Journal for Theory, History and Foundations of Science*, 36, 1: 49-62.

Karam, Ricardo, 2020: "Why are complex numbers needed in quantum mechanics? Some answers for the introductory level", in *American Journal of Physics*, 88: 39-45.

Knowles, Robert, 2015, "Heavy Duty Platonism", in *Erkenntnis*, 80, 6: 1255-1270.

Lewis, David, 1983, "Extrinsic Properties", in *Philosophical Studies*, 44, 197-200.

—, 1988, "Statements partly about observation", in *Philosophical Papers*, 17, 1: 1-31.

Liggins, David, 2016, "Grounding and the indispensability argument", in *Synthese*, 193, 2: 531-548.

Linsky, Bernard, Edward N. Zalta, 1995, "Naturalized Platonism versus Platonized naturalism", in *The Journal of Philosophy*, 92: 525-555.

Meinong, Alexius, 1904, "Über Gegenstandstheorie", Meinong, Alexius (eds). *Untersuchungen zur Gegenstandstheorie und Psychologie*, J.A. Barth, Leipzig: 1-51; Eng. tr. 1960, "The Theory of Objects", in Chisholm, ed., *Realism and the Background of Phenomenology*, The Free press-Collier-Macmillan, New York & London: 76–117.

Nodelman, Uri, Edward N. Zalta, 2014, "Foundations for Mathematical Structuralism", in *Mind*, 123, 489: 39-78.

Panza, Marco, Andrea Sereni, 2013, *Plato's Problem. An Introduction on Mathematical Platonism*, Palgrave Macmillan, London.

Parent, Ted, 2020, "Ontological Commitment and Quantifiers", in R. Bliss, J.T.M. Miller (eds), *The Routledge Handbook of Metametaphysics*, Routledge, Abingdon.

Parsons, Charles, 1990, "The Structuralist View of Mathematical Objects", in *Synthese*, 84: 303-346.

Parsons, Terence, 1980, "*Nonexistent objects*", Yale University Press, New Haven and London.

Plebani, Matteo, 2018, "The indispensability argument and the nature of mathematical objects", in *Theoria: An International Journal for Theory, History and Foundations of Science*, 33, 2: 249-263.

Priest, Graham, 2003, "Meinongianism and the philosophy of mathematics", in *Philosophia Mathematica*, 11, 1: 3-15.

—, 2005, *Towards Non-Being: The Logic and Metaphysics of Intentionality*, Oxford University Press, Oxford.

Quine, Willard Van Orman, 1948, "On What There Is", in *Review of Metaphysics*, 2: 21-38.

Routley, Richard, 1980, *Exploring Meinong's Jungle and Beyond*, Australian National University, RSSS, Canberra.

Russell, Bertrand, 1903, *Principles of Mathematics*, Cambridge University Press, Cambridge.

—, 1905a, "On denoting", in *Mind,* 14: 479-493.

—, 1905b, "Review of A. Meinong, Untersuchungen zur Gegenstandtheorie und Psychologie", in *Mind*, 14: 530-538.

Schaffer, Jonathan, 2009, "On what grounds what", in David Manley, David J. Chalmers, Ryan Wasserman (eds.), *Metametaphysics: New Essays on the Foundations of*

Ontology, Oxford University Press, Oxford: 347-383.

Sendłak, Maciej, 2023, "From the Meinongian Point of View", in *Logic and Logical Philosophy*, 32, 2: 243-251.

Shapiro, Stewart, 1997, *Philosophy of Mathematics: Structure and Ontology*, Oxford University Press, New York.

Van Inwagen, Peter, 1998, "Meta-ontology", in *Erkenntnis*, 48: 233-50.

Varzi, Achille C., 2005, *Ontologia*. Laterza, Rome.

Yablo, Stephen, 2012, "Explanation, extrapolation, and existence", in *Mind*, 121, 484: 1007-1029.

—, 2014, *Aboutness*, Princeton University Press, Princeton.

Zalta, Edward, 1983, *Abstract objects: An introduction to axiomatic metaphysics*, D. Reidel, Dodrecht.

—, 1988, *Intensional Logic and the Metaphysics of Intentionality*. MIT Press, Cambridge, MA.